8. Sketch the curve $y = x^3 + x$

2074 Solution:

here, $f(x) = x^3 + x$.

i. Domain:

clearly, $f(x)$ is defined $\forall\ x \in (-\infty, \infty)$.

$\therefore$ Domain $= (-\infty, \infty)$

ii. Intercepts:

put $x = 0$, $y = 0$.

put $y = 0$, ~~$x = 0$~~ $x^3 + x = 0$

$\Rightarrow x(x^2 + 1) = 0$

$\therefore x = 0$, $x^2 = -1$ (imaginary)

So, the curve passes through $(0,0)$.

iii. Symmetricity:

put $x = -x$,

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -(x^3 + x)$$

$\therefore f(-x) = -f(x)$

So, it is symmetric about an origin.

iv. Asymptotes:

Horizontal asymptotes:

$\lim_{x \to \infty} f(x)$ or $\lim_{x \to -\infty} f(x) = \mathcal{I}$ (doesn't exist)

Vertical asymptotes:

$\lim_{x \to a} f(x) = \infty$ (doesn't exist)

$\therefore$ No asymptotes.

v. Increasing and decreasing :

$$f'(x) = 3x^2 + 1 .$$

At critical point,

$$3x^2 + 1 = 0$$

or, $\cancel{3}x^2 = -\frac{1}{3}$

$\therefore x = \sqrt{-\frac{1}{3}}$ (imaginary)

Also,

$$f'(x) \to \infty$$

or, $3x^2 + 1 = \frac{1}{0}$

or, $0 = 1$ (false)

Hence, there doesn't exist any interval of increasing and decreasing.

vi. Maxima / Minima :

No maxima or minima.

vii. Concavity :

$$f''(x) = 6x$$

At point of inflection,

$$6x = 0$$

$$\therefore x = 0$$

Also,

$$f''(x) \to \infty .$$

or, $6x = \frac{1}{0}$

$0 = 1$ (false)

Interval	Sign of $f'(x)$	Nature of $f'(x)$
$(-\infty, 0)$	$-ve$	concave downward
$(0, \infty)$	$+ve$	concave upward.

Using above informations to sketch the graph;